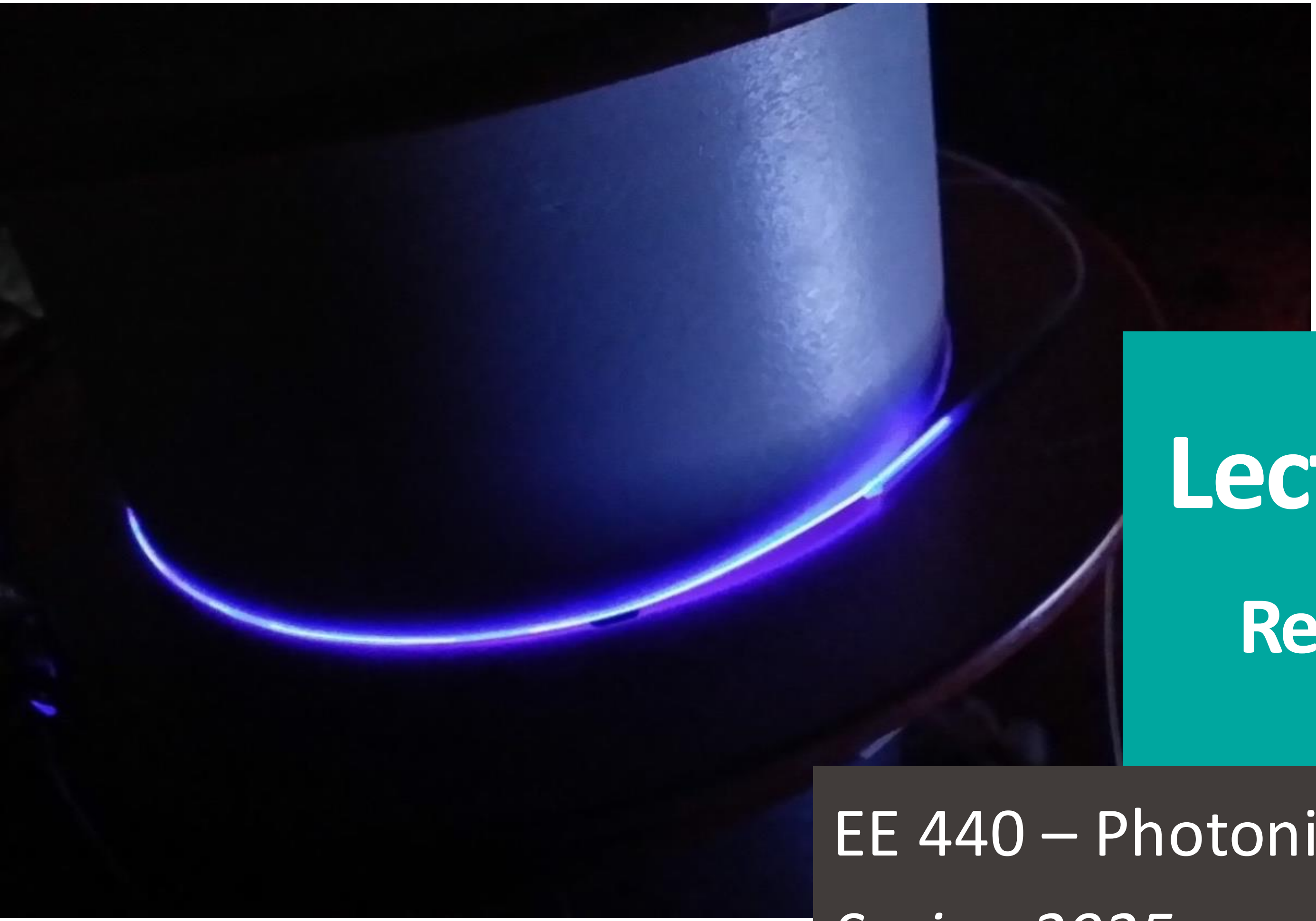
EPFL



Lecture 9

Receivers and noise

EE 440 – Photonic systems and technology

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Ecole Polytechnique Fédérale de Lausanne

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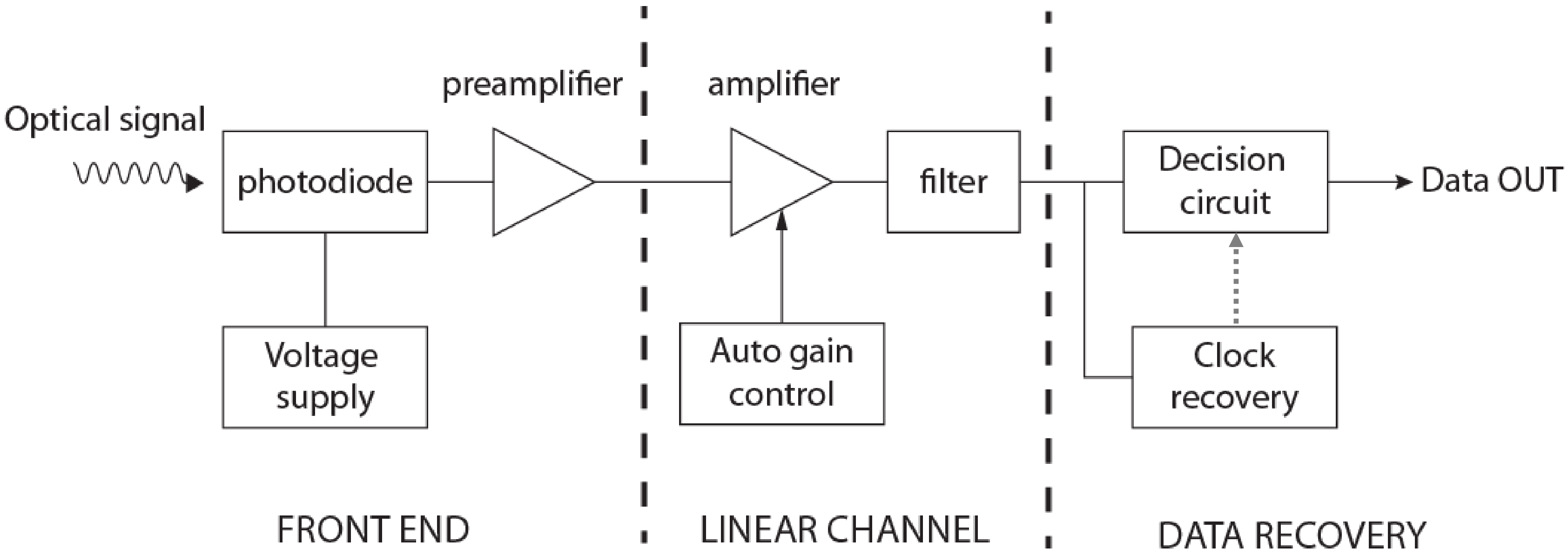
Other components in the receiver

Receiver components

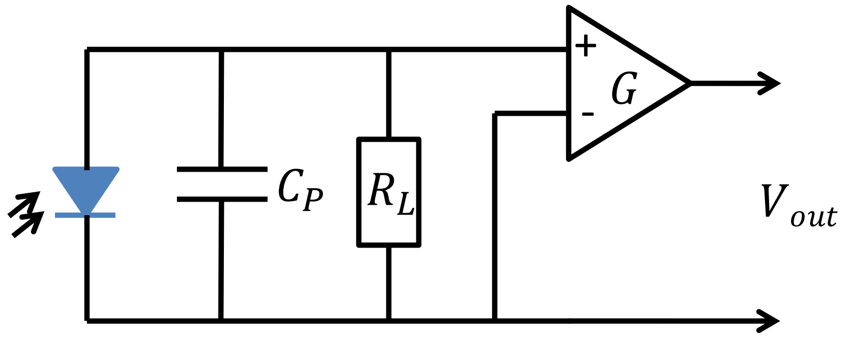
Design of optical receiver varies depending on modulation format
Digital receivers have three components:



Receiver front ends

Consists of photodiode + preamplifier used to amplify the electrical signal for further processing

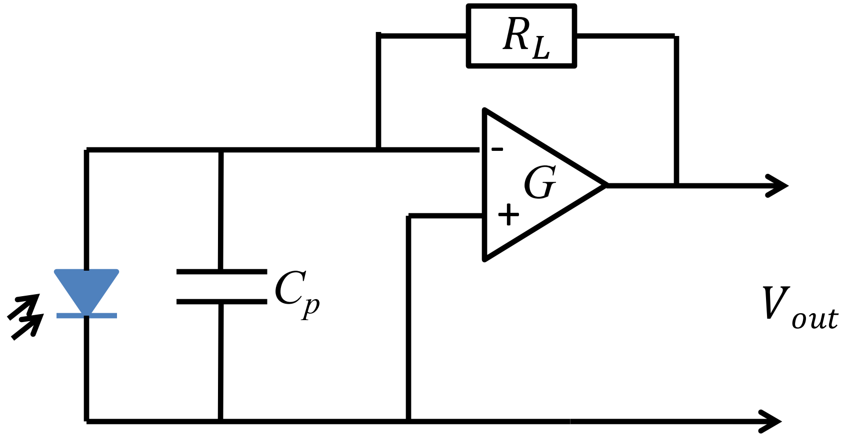
High impedance front end



- Increase input voltage to the preamplifier with by increasing R_L
  - Increased sensitivity, better noise
  - *Decreased bandwidth* due to large RC constant

$$\Delta f \approx \frac{1}{2\pi R_L C_P}$$

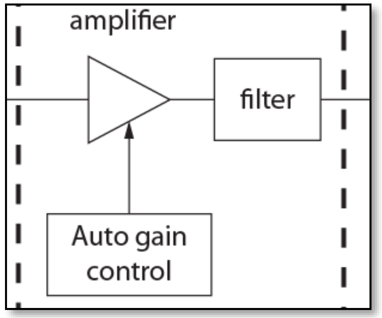
Trans-impedance front end



- Use R_L as feedback resistor with inverting amplifier: reduces effective impedance by factor G
  - High sensitivity, low noise and high bandwidth
  - Main problem *stability* due to the feedback loop

$$\Delta f \approx \frac{G}{2\pi R_L C_P}$$

Linear channel



The linear channel consists of:

- A high gain amplifier with automatic gain control
  - Constant average output voltage regardless of the input (within limits)
- A low pass filter with bandwidth chosen to
  - Reject noise outside of the signal bandwidth and avoid inter-symbol interference (ISI)

The best situation is when only the filter limits the overall bandwidth of the receiver

Given the linear channel transfer function $H_T(f)$, the output voltage transfer function is then:

$$H_{out}(f) = H_T(f)H_P(f)$$

Transfer function of linear channel
Most of the time dominated by filter

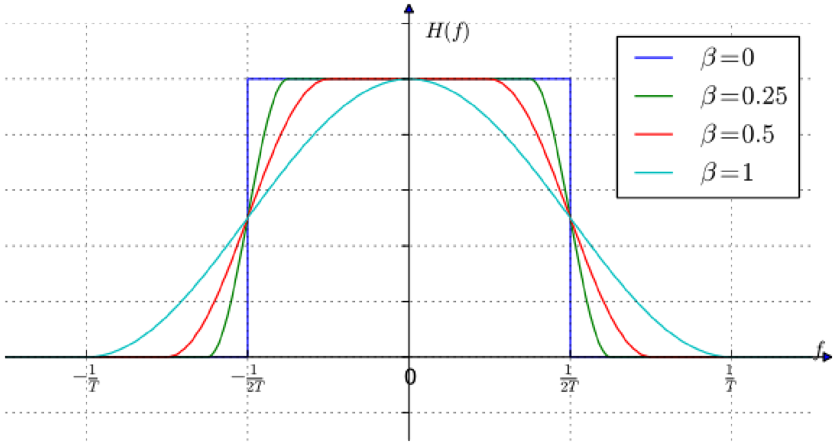
Known normalized spectral function of input photocurrent pulse

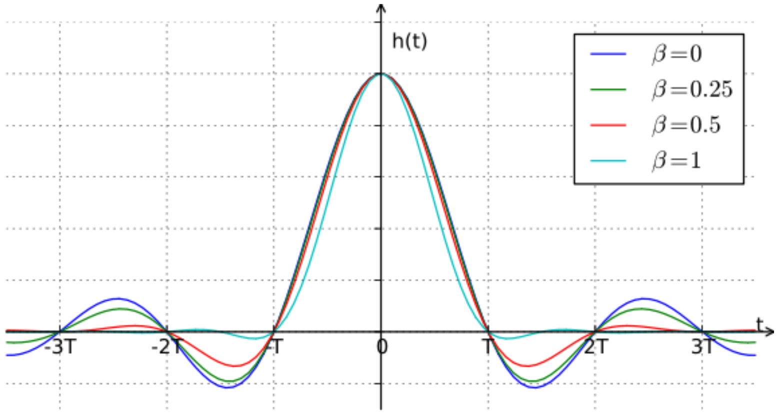
Linear channel

In order to minimize ISI want a raised cosine type response at the output of the linear channel

- Response is maximized at the decision instance $t = 0$
- Response is minimized (= 0) at other sampling times (i.e. zero crossing)

$$H_{out}(f) = \begin{cases} 1 & |f| \leq B\frac{1-\beta}{2} \\ \frac{1}{2}\left[1+\cos\left(\frac{\pi}{\beta B}\left[|f| - B\frac{1-\beta}{2}\right]\right)\right], & B\frac{1-\beta}{2} < |f| < B\frac{1+\beta}{2} \\ 0 & \text{otherwise} \end{cases}$$





Linear channel - example

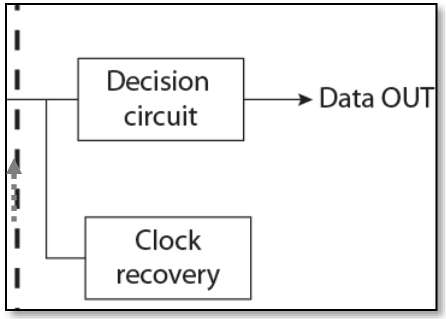
What should be the transfer function of the linear channel if we are processing NRZ data at a bit rate B, assuming rectangular input pulse shape and a filter with $\beta = 1$

- Square NRZ pulses : $H_p(f) = \mathrm{sinc}\left(\frac{\pi f}{B}\right) = \frac{B}{\pi f}\sin\left(\frac{\pi f}{B}\right)$
- Want $H_{out}(f) = \frac{1}{2}\left(1 + \cos\left(\frac{\pi}{B}f\right)\right), |f| < B$
- Therefore: $H_T(f) = \frac{H_{out}(f)}{H_P(t)} = \frac{\pi f}{2B}\frac{\left(1 + \cos\left(\frac{\pi}{B}f\right)\right)}{\sin\left(\frac{\pi f}{B}\right)} = \frac{\pi f}{2B}\frac{2\cos^2\left(\frac{\pi}{2B}f\right)}{2\sin\left(\frac{\pi f}{2B}\right)\cos\left(\frac{\pi f}{2B}\right)}$

$$H_T(f) = \frac{\pi f}{2B}\cot\left(\frac{\pi}{2B}f\right), |f| < B$$

Data recovery

Consists of a clock recovery unit and decision circuit

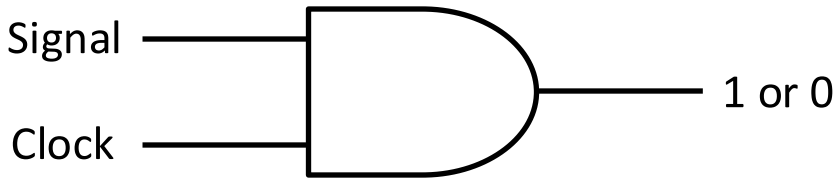


Clock recovery unit isolates a spectral component at $f = B$ from the received signal

- Provides information to the decision circuit
- Helps synchronize the decision process

Simple decision circuit

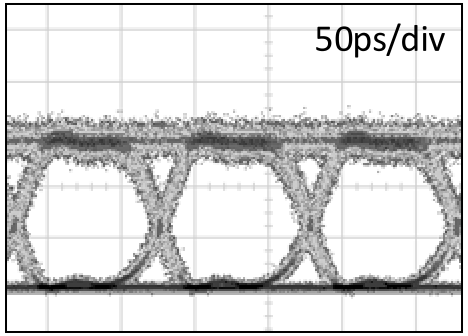
- For example, sample taken at the peak of the clock

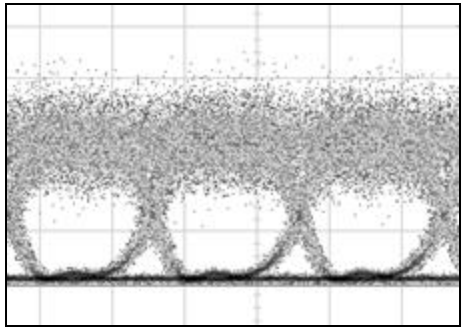


Eye patterns

To measure the characteristics of a received data signal:

- Use a clock signal to trigger an oscilloscope and feed the random data to the scope input
- The resulting signal is synchronously superimposed
- All possible data patterns are shown simultaneously, giving the appearance of an ‘eye’

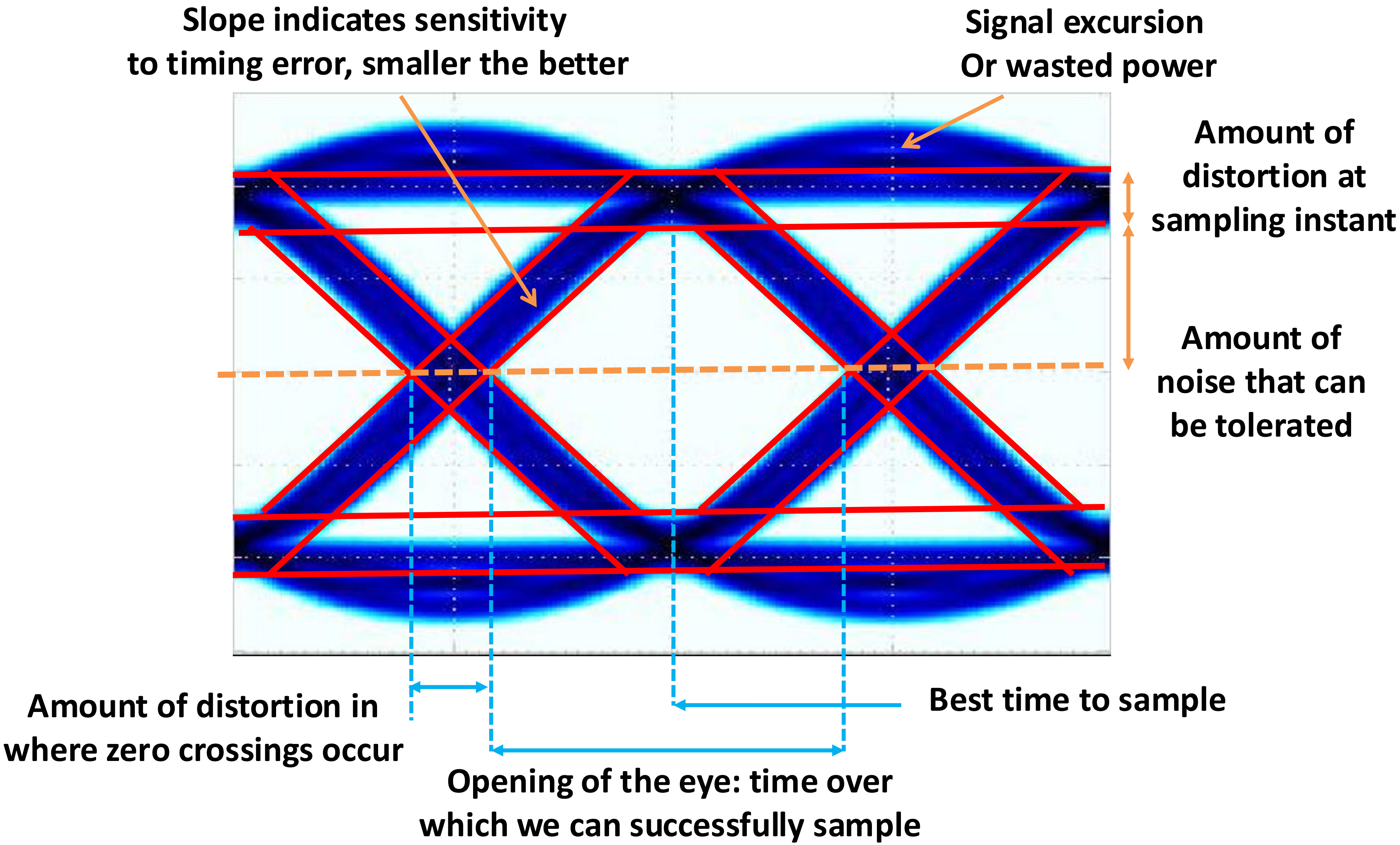




Provides information on :
- Signal to noise ratio (SNR)
- Amplitude noise
- Timing jitter
- Rise and fall time of detection circuit

Best sampling time/threshold is the maximum opening of the eye

Eye diagram interpretation



Receiver noise

Noise due to receiver

Optical receivers convert optical power into electric current

$$i_P(t) = RP_{in}(t)$$

i_P is the *average current*: total current actually contains noise even when P_{in} is deterministic (constant)

The fundamental noise mechanisms are shot noise and thermal noise

$$I(t) = i_P + i_S(t) + i_T(t)$$

Average current (i_P)

Shot noise current ($i_S(t)$)

Thermal noise current ($i_T(t)$)

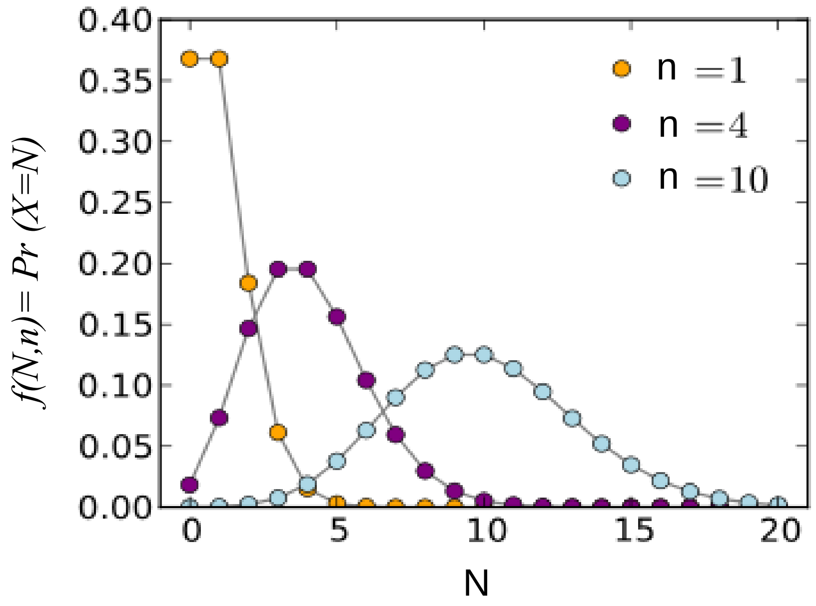
Shot noise

Electric current consists of a *stream of electron* that are generated at *random times*

Constant illumination of PD leads to generation of photo-electrons exhibiting a Poisson distribution

$$f(N,n) = \frac{e^{-n} n^N}{N!}$$

n: expected number of occurrence in given time interval
N: number of occurrences



Mean = n
Variance = n
Standard deviation = $\sqrt{variance} = \sqrt{n}$

For large mean values, distribution starts looking Gaussian.

Shot noise

Shot noise current $i_s(t)$ is a random process.

- At a particular instant in time t', the value of $i_s(t')$ is a random variable. In our case approximated by a zero mean Gaussian.
- Time fluctuations in $i_s(t)$ can be characterized by its averaged frequency spectrum called the power spectral density $S_s(f)$

For Gaussian white noise: $S_S(f) = qi_p$

We can show, using the Wiener-Khinchin theorem that shot noise variance is given by:

$$\sigma_S^2 = 2qi_p\Delta f$$

Dark current contribution

Recall that even without illumination a dark current i_d flows

- Current generated by the photodiode which is independent of light
- Also contributes to the generation of shot noise.

The overall variance of the shot noise contribution is therefore:

$$\sigma_S^2 = 2q\left(i_p + i_d\right)\Delta f$$

Total shot noise variance

- The quantity σ_s is the root mean square (RMS) value of the noise current induced by shot noise

Thermal noise

At finite temperature, electrons move randomly in resistor: *random thermal motions* lead to fluctuating current

Thermal current $i_T(t)$

- Stationary Gaussian random process,
- Nearly white: frequency independent up to about 1 THz
- Signal independent
- Thermal noise is also called Johnson or Nyquist noise

Power spectral density depends on front end load resistor R_L: $S_S(f) = \frac{2k_B T}{R_L}$

We can show that thermal noise variance is given by:

$$\sigma_T^2 = \frac{4k_B T \Delta f}{R_L} F_n$$

Total thermal noise variance

F_n :Electrical amplifier noise figure

Signal to noise ratio for p-i-n

Since the different noise sources are uncorrelated, total noise variance:

$$\sigma^2 = \sigma_S^2 + \sigma_T^2 = 2q(i_p + i_d)\Delta f + \frac{4k_B T \Delta f}{R_L} F_n$$

The signal to noise ratio (SNR) of an electrical signal is defined as:

$$SNR = \frac{average\ signal\ power}{total\ noise\ variance} = \frac{i_p^2}{\sigma^2}$$

$$SNR = \frac{R^2 P_{in}^2}{2q(i_p + i_d)\Delta f + \frac{4k_B T \Delta f}{R_L} F_n}$$

Thermal and shot noise limits

Thermal noise limit $\sigma_T^2 \gg \sigma_S^2$

$$SNR_{T,\mathrm{pin}} = \frac{R_L R^2 P_{in}^2}{4 k_B T \Delta f F_n}$$

Shot noise limit $\sigma_S^2 \gg \sigma_T^2$

- Dark current can be neglected

$$SNR_{S,\mathrm{pin}} = \frac{R P_{in}}{2 q \Delta f}$$

We note

- Different scaling with input powers in the two limits
- Thermal noise dominates at low input powers
- Shot noise dominates at high input powers

Noise limits - example

We have a receiver with $R = 0.95\ \mathrm{A/W}$. We can use two different load resistance of $R_L = 50\ \Omega$ and $R_L = 100\ \mathrm{k}\Omega$.

What is the signal power required to obtain shot noise limited operation at room temperature?

If the bandwidth of our receiver is limited by the RC time constant, which load resistance would you choose for highly sensitive and which one for high-speed detection ?

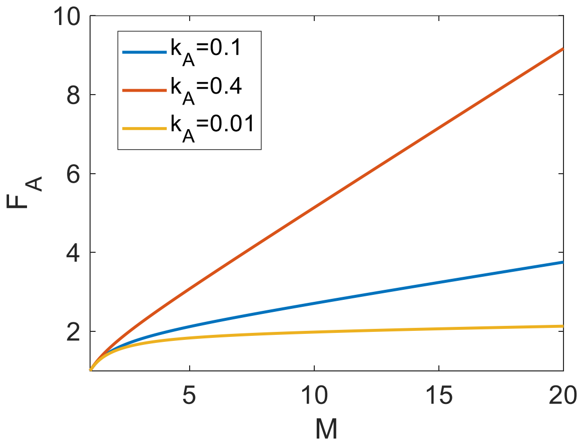
Noise in APD receivers

Thermal noise remains the same since originates in electrical components not part of the APD.

Shot noise will be affected as secondary electron-hole pairs are also generated at random times.

$$\sigma_S^2 = 2qM^2F_A(RP_{IN} + i_d)\Delta f$$

With $F_A(M) = k_A M + (1 - k_A)(2 - {}^1/_M)$ the excess noise factor



Signal to noise ratio:

$$SNR = \frac{(MRP_{IN})^2}{2qM^2F_A(RP_{IN} + i_d)\Delta f + \frac{4k_BT\Delta f}{R_L}F_n}$$

Shot noise limit with APD

$$SNR_{S,\mathrm{APD}} = \frac{(MRP_{IN})^2}{2qM^2F_A(RP_{IN} + i_d)\Delta f + \frac{4k_BT\Delta f}{R_L}F_n}$$

$$SNR_{S,\mathrm{APD}} = \frac{RP_{IN}}{2qF_A\Delta f} = \frac{SNR_{S,\mathrm{pin}}}{F_A}$$

$$SNR_{S,\mathrm{APD}} = \frac{SNR_{S,\mathrm{pin}}}{F_A}$$

APD avalanche noise *degrades* SNR compared to p-i-n diode

Thermal noise limit with APD

$$SNR_{T,APD} = \frac{(MRP_{IN})^2}{\cancel{2qM^2F_A(RP_{IN}+i_d)\Delta f} + \frac{4k_BT\Delta f}{R_L}F_n}$$

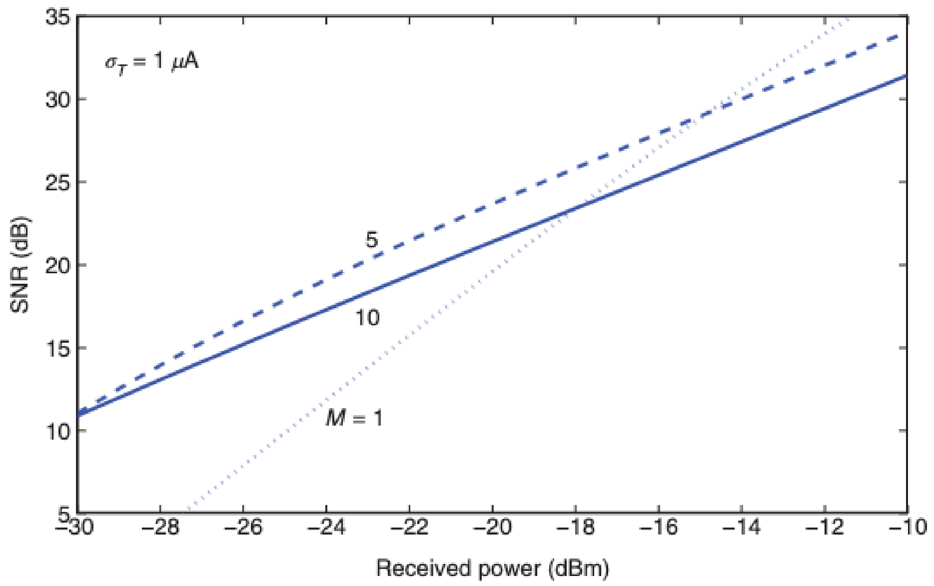
$$SNR_{T,\mathrm{APD}} = \frac{R_LM^2(RP_{IN})^2}{4k_BT\Delta f\,F_n}$$

$$SNR_{T,\mathrm{APD}} = M^2 SNR_{T,\mathrm{pin}}$$

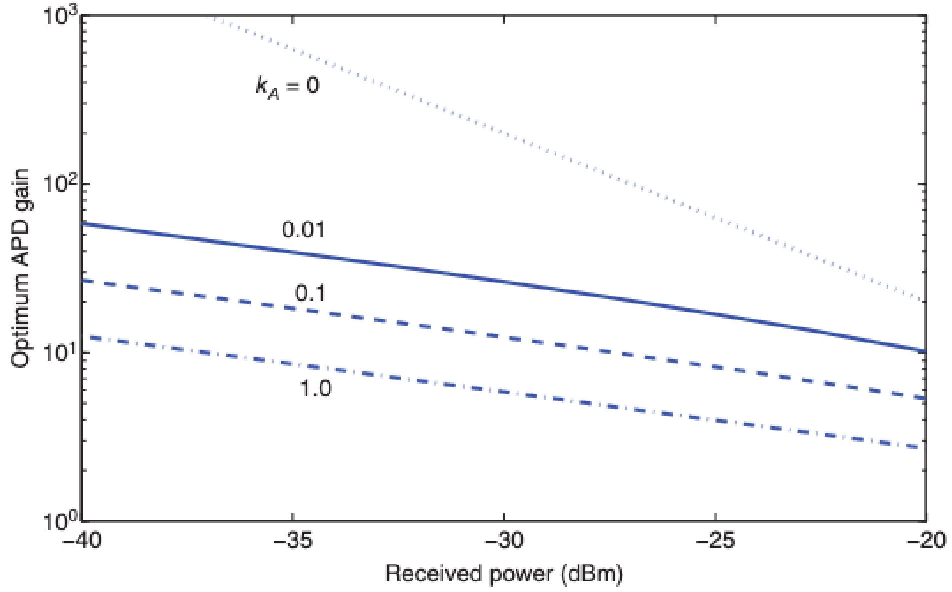
APD gain swamps out thermal noise, *increases* the SNR.

The APD versus the p-i-n

Example of the SNR (Δf = 30 GHz) for p-i-n receiver and APD receiver



- APD is best at low power
- p-i-n is best at high power



There is optimum value for M

$$M_{\text{opt}} = \left[\frac{4k_B T F_n}{k_A q R_L (RP_{in} + i_d)}\right]^{1/3}$$

Depends on ionization factor k_A

- Highest M_{opt} ~100 for silicon APD
- Highest M_{opt} ~10 for InGaAs APD